DISCREPANCY BETWEEN THEORETICAL AND EXPERIMENTAL DATA ON NONSTEADY HEAT CONDUCTION IN DISPERSE SYSTEMS

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An attempt is made quantitatively to estimate the feasibility of applying the working differential equation for a continuous medium to a disperse system.

Results are also given for the experimental cooling of a cylindrical calorimeter in a bed of glass beads 0.1, 0.35, 1.0, and 5 mm in diameter in the time range 0.1 sec ≤ τ ≤ 100 sec.

The mechanism of nonsteady heat conduction was considered in previous articles [1, 2], and a system of differential equations was proposed which takes into account the finite temperature-perturbation relaxation rate in disperse systems. After these articles were published, Rubinshtein kindly sent us a copy of his own paper [3], where it was revealed that the problem of theoretically describing heat propagation in heterogeneous media, with allowance for relaxation processes, was solved by a similar method more than 30 years ago. For the case of a bicomponent system in an electromagnetic circuit, this problem was posed by Arkad'ev and solved by Tikhonov [4].

Studying the dynamics of sorption processes, Tikhonov found that it is possible to apply the sorption equations he derived to the problem of heat propagation in soil; unfortunately, however, this study was not published. Finally, Rubinshtein, on the basis of a statistical analysis of the effect, obtained and solved the corresponding system of equations. At present, the absence of the required experimental data and the limited experimental means available do not permit an experimental evaluation of the predicted inhibition of the growth of a temperature field in a disperse medium which should result from relaxation of local-temperature differences of the medium and particles. Apparently, this can be explained by the fact that in the ongoing development of the theory of heat conduction in disperse systems, a consequence of the widespread introduction of various industrial processes and the use of new advanced filler- and binder-based materials, the proposed approach has been neglected and the differential equation for the heat conduction of a continuous medium has been used universally. As far as we know, the conclusion that the differential equation of a continuous medium cannot, under certain conditions, be used to describe the nonsteady heat conduction of a disperse system was first made in [5] on the basis of experimental data.

In this article, an attempt is made quantitatively to evaluate the possibility of applying the working differential equation of a continuous medium to a disperse system. To make it easier to compare the experimental and theoretical data, we determine the time change in the relative temperature of an infinite cylindrical calorimeter when it contacts an unbounded disperse medium (assumed in the calculation to be a continuous body).

The solution of the differential equation

$$\frac{\partial \vartheta_1}{\partial \tau} = a\left(\frac{\partial^2 \vartheta_1}{\partial r^2} + \frac{1}{r}\,\frac{\partial \vartheta_1}{\partial r}\right), \tag{1}$$

under the boundary conditions

$$\vartheta_1|_{\tau=0} = 0,\ \ \vartheta_2|_{\tau=0} = \vartheta_0,$$

$$\vartheta_1|_{r=r_0} = \vartheta_2|_{r=r_0},\ \ \left.\frac{\partial \vartheta_1}{\partial r}\right|_{r\to\infty} = 0,$$

$$r_0\left.\frac{\partial \vartheta_2}{\partial \tau}\right|_{r=r_0} = a\beta\left.\frac{\partial \vartheta_1}{\partial r}\right|_{r=r_0},$$

is known [6]. This solution has the form

$$\frac{\vartheta_2}{\vartheta_0} = \frac{4\beta}{\pi^2}\int\limits_0^\infty \exp\left(-\frac{a\tau u^2}{r_0^2}\right)\frac{du}{u\Delta(u)}, \tag{2}$$

where

$$\Delta(u) = [uJ_0(u) - \beta J_1(u)]^2 + [uY_0(u) - \beta Y_1(u)]^2.$$

The improper integral on the right-hand side of (2) is evaluated with a quadrature formula which provides the highest algebraic accuracy [7].

Equation (2) cannot be used when the parameter has a value corresponding in our case to $\tau \le 20$ sec. In the representation region, relation (2) has the form

$$\frac{\bar{\vartheta}_2}{\vartheta_0} = \frac{1}{aq^2\left[1 + \frac{A}{q}\,\frac{K_1(r_0q)}{K_0(r_0q)}\right]}. \tag{3}$$

An expression suitable for numerical calculations at small times is obtained by asymptotically expanding the Bessel functions in the representation region; this can be done for large $q = (s/a)^{1/2}$. This obtained expression has the form

$$\frac{\vartheta_2}{\vartheta_0} = 1 + A_1\tau^{1/2} + A_2\tau + A_3\tau^{3/2} + A_4\tau^2 + \ldots,$$

$$A_1 = \frac{2\beta}{r_0}\sqrt{\frac{a}{\pi}};\ \ A_2 = -\frac{\beta(1-2\beta)a}{2r_0^2},$$

$$A_3 = \frac{\beta(1+8\beta-8\beta^2)a}{6r_0^3}\sqrt{\frac{a}{\pi}},$$

$$A_4 = -\frac{\beta(1-2\beta-4\beta^2+8\beta^3)a^2}{16r_0^4}. \tag{4}$$

The quantity λ_{eff} is calculated from the formula in [12]; values for the thermal conductivity of the par-

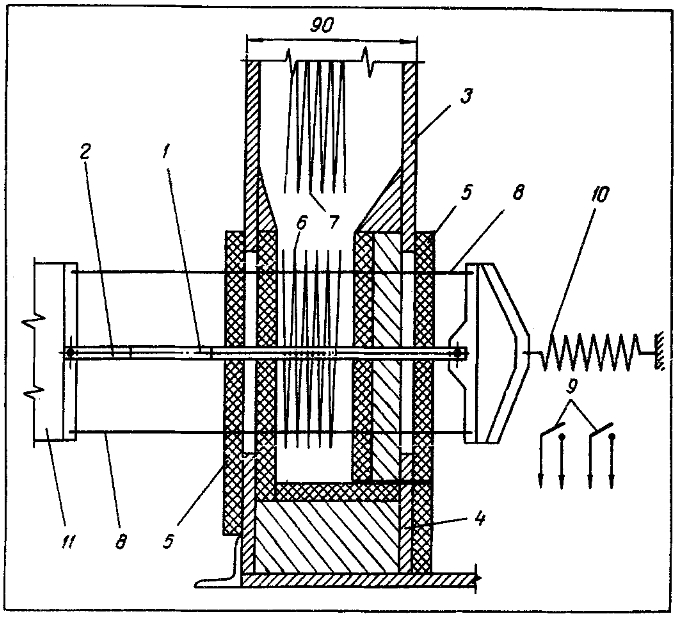

Fig. 1. Schematic view of the experimental device: 1) calorimeter; 2) ebonite holder; 3) wall of column (plexiglas); 4) removable base; 5) baffle (porolon); 6) compensation arm of the measurement bridge; 7) temperature control transducer at the wall; 8) rod (steel cable); 9) end switch (device for determining the time the calorimeter enters the column); 10) spring; 11) conveyer belt.

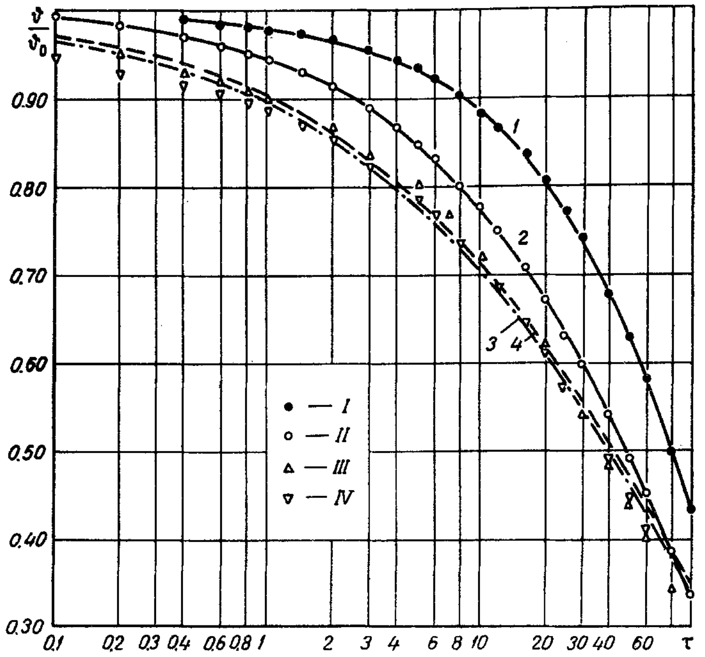

Fig. 2. Change in relative temperature of calorimeter as a function of time (τ in sec): 1) cooling in a bed of glass beads 5 mm in diameter; 2) 1 mm in diameter; 3, 4) calculated using formulas (4) and (2) for particles 1 and 5 mm in diameter; I—glass beads 5 mm in diameter; II—1 mm; III—0.35 mm; IV—0.1 mm.

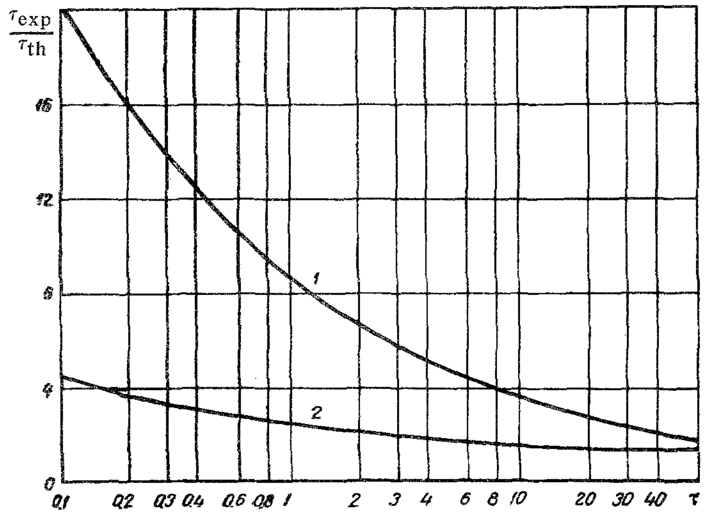

Fig. 3. Ratio of experimental and theoretical temperature equilibrium times as a function of time (τ in sec); 1) for particles 5 mm in diameter; 2) 1 mm.

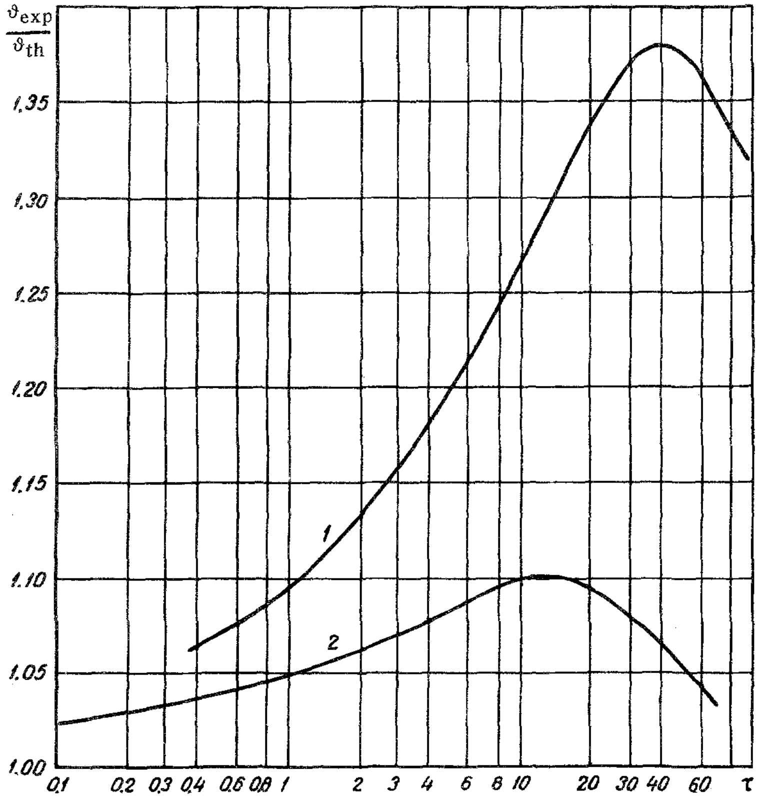

Fig. 4. Ratio of the actual temperature to the calculated temperature as a function of time (τ in sec): 1) particles 5 mm diameter; 2) 1 mm.

ticles, $\lambda_s = 1.05$ W/m · deg, and specific heat $c_s = 0.755$ kJ/kg · deg are taken from the data in [5]. The porosity is found from the formula $\varepsilon = 1 - \rho_m/\rho_s$; here, the quantities ρ_m and ρ_s are determined using the standard technique in [8]. The material was weighed in a vessel with inner diameter of 70 mm.

Existing experimental data on nonsteady heat transfer in a bed of disperse material [5, 10, 11, etc.] were obtained under conditions where the experimental errors cannot be enumerated and systematized, because of the complexity of allowing for heat loss and other secondary effects (the rotation of particles moving toward the surface, the immersion of the moving calorimeter in the wake of particles which have previously been heated, the detached flow of the bed around the calorimeter). These drawbacks all contribute to unreliable experimental data, both for long and short contact times. For this reason, the method developed in [13] is used in the present study for measuring the heat-transfer coefficients in nonstationary processes with $\tau \geq 0.001$ sec. The calorimeter is a solid copper cylinder 6 mm in diameter and 40 mm long; a copper wire (resistance thermometer) 0.02 mm in diameter is fitted into a helical groove 0.05 mm deep on the surface of the calorimeter. The resistance thermometer is coupled to the input of a TA-5 tensometric amplifier as (an ordinary wire-resistance strain gauge [14]). In the second (free) arm of the measurement bridge of the TA-5, there is a compensation resistance thermometer; this thermometer is always found at that point in the bed where the disperse material comes into contact with the wall.

We can show that when the resistance thermometers (basic and compensation) are connected in such a manner, the signal in the measurement diagonal of the bridge is proportional to the difference in resistance (temperature) of the two thermometers, i.e., the excess temperature of the calorimeter is measured. The unbalance resulting from the change in temperature is amplified in the measurement diagonal of the bridge by the TA-5 tensometric amplifier and recorded on photographic paper by a 9SO-1F2 oscillograph.

Figure 1 shows a schematic diagram of the experimental setup. The device consists of: a) a rectangular column made from plastic 6 mm thick; the sides of this column are 90 × 120 mm and are heat-insulated on the inside by a layer of foam plastic 16 mm thick and porolon 10 mm thick. Rectangular ports 100 × × 10 mm are cut into the major sides of the column (on the axis); these columns have baffles to prevent the material from leaving the column and are provided to allow the calorimeter and rod to pass through the column; b) a spring mechanism with a locking device to admit and remove the calorimeter from the column; c) an air heater with a nozzle.

A typical experiment consisted of the following. A column containing a calorimeter was filled to some particular level by a standard disperse material (glass beads 0.1, 0.35, 1, or 5 mm in diameter) and held in this state for a few hours. Approximately one hour after the measurement equipment was inserted, statistical calibration was performed; as a result, the deviation of the beam on the oscillograph screen was obtained as a function of the calorimeter heating temperature. Calibration was performed by making 0.3-ohm · A changes in the resistance of the MSR-58 box which was coupled in series with the resistance thermometer of the calorimeter; this corresponded to a temperature increase of about 6° C. Such a small increase in temperature was chosen so that the effect of the change in thermophysical properties of the material during the experiment was minimized. To perform static calibration, it was determined that the beam deflection was proportional to the change in resistance ΔR. After calibration, the cylinder with the locking device was opened and the calorimeter was disconnected from the column; the air leaving the nozzle of the air heater was then used to heat the calorimeter to a temperature approximately 6° C greater than the temperature of the layer. After the hot-air jet ceased and after a calibrated line corresponding to a given increase in temperature was reached using the "movable light spot" of a vibrator, the calorimeter was inserted into the layer with the aid of the spring mechanism and the cooling process was recorded.

Calculations were performed with the oscillograms for the time change in transducer resistance. The sensitivity of the circuit varied from about 1 to 3° C over the entire width of the recording photographic paper (10 cm). The temperature was held constant at the layer boundary by visual observation of the needle deflection of the F116/1 microvoltmeter (inserted into the diagonal of the bridge circuit) and by resistance thermometers in the arms; these thermometers were placed on the layer boundary and on the inner wall of the column directly above the layer (Fig. 1). The parameters of the circuit made it possible reliably to register 0.2° C changes in temperature on the inner boundary of the layer. The results obtained from processing the experimental data are given in Figs. 2, 3, and 4.

The solid lines in Fig. 2 show the time change in calorimeter temperature for a bed of glass beads 1 and 5 mm in diameter. For small τ, it is not possible to assume that there is no temperature gradient for the calorimeter. Calculations show that the values of τ [9] for which the temperature gradient must be taken into account lie within the limits of the times of practical interest.

It follows from Fig. 2 that for the studied time range, the relative temperatures of a cylinder cooled in beds of particles 0.1 and 0.35 mm in diameter are almost equivalent to the theoretical values (for smaller times, there are also deviations from the theoretical curves for these particle diameters). For particles 1 and 5 mm in diameter, the discrepancy becomes even greater.

As is clear from Figs. 3 and 4, the use of the theoretical relationship can lead to significant error (by a factor of 2–20) when estimating the time required for a surface to reach a given temperature, and may be as great as 40% when estimating the magnitude of the temperature.

There has been very little study performed on the features of heat transfer by disperse layers for small times. The given measurement results and their comparison with the theoretical data shows how many gaps there are in the theory of the heat conduction of disperse systems. The absence of data on the relaxation rate of the transfer potentials imposes great restrictions on a theoretical analysis of nonsteady heat conduction in a bed of disperse material and on a more general formulation for an analysis of the kinetics of energy transfer and matter transport in multicomponent bodies. It is only known that for some fairly large times, the theoretical and experimental data coincide.

It is very important to determine to what extent the differential equation for the heat conduction of continuous media can be applied to a disperse system and to extend the limits by adding certain assumptions to the theoretical model [1–3]. The results of such an attempt will be discussed in a subsequent article.

NOTATION

ϑ_1 is the temperature of medium; ϑ_2 is the calorimeter temperature; ϑ_0 is the initial calorimeter temperature; r_0 is the calorimeter radius; c_s is the solid-phase specific heat; $c_s(1-\varepsilon)$ is the specific heat of the medium; ε is the porosity; λ_s is the thermal conductivity of the solid phase; λ_{eff} is the effective thermal conductivity of the medium; ρ_s is the solid-phase density; ρ_m is the density of the medium; a is the thermal diffusivity of the medium; τ is the time; $\beta = 2\pi r_0^2 c_s \rho_s (1-\varepsilon)/C$ is a parameter equal to twice the ratio of the specific heat of an equivalent volume of medium to the specific heat of an ideal conductor (calorimeter); C is the total specific heat of the calorimeter per unit length; $J_0(u)$ is a zero-order Bessel function of the first kind; $J_1(u)$ is a first-order Bessel function of the first kind; $Y_0(u)$ is a zero-order Bessel function of the second kind; $Y_1(u)$ is a first-order Bessel function of the second kind; $K_0(u)$ is a modified zero-order Bessel function of the second kind; $K_1(u)$ is a modified first-order Bessel function of the second kind.

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